BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 67 (71), Numărul 4, 2021
Secţia
CONSTRUCŢII DE MAŞINI

MODELS FOR THE INFLUENCE OF TRANSMISSION ERRORS AND HOBBING FORCES UPON A COMPLEX GEAR DRIVE

BY

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Received: September 10, 2021
Accepted for publication: December 14, 2021

Abstract. The paper describes a theoretical research approach that performs a dynamic study upon a complex gear transmission. The transmission errors of the gears from the considered complex gear drive are considered when kinematics is analyzed and presented. The cutting forces from a gear hobbing process, possibly driven by such a complex gear transmission, are considered when parameters of the dynamic model are mathematically evaluated and presented. The proposed model development includes the obtaining of the differential equation for kinetic energy. Some discussions are also made regarding the studied complex mechanism. Practically, mathematical models for the influence of transmission errors and hobbing forces upon the considered complex gear drive are proposed as deliverable results of the study. Final conclusions are also included in the paper. The aspects discussed in the paper are presenting utility for the design of such mechanisms or for calculating the parameters of some gear hobbing processes.

Keywords: complex gear drive; kinematic model; dynamic model; transmission errors; gear hobbing forces.

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1. Introduction

The present paper is desired to be a continuation and a development of a previous paper (Merticaru and Merticaru, 2020).

In that paper, the authors were studying the dynamics of a complex gear transmission which could be used for gear cutting on a hobbing machine. The kinematics and the parameters of a simple dynamic model are presented. Only the overall dynamic parameters of the gear transmission are taken into account. Other parameters like elasticity of kinematic elements, meshing stiffness, friction, transmission errors or the expression of the technological forces are neglected.

In the present paper, the kinematics and the dynamics of the complex gear transmission is completed and developed by taking into account the transmission errors of the gears and the cutting forces that appear during hobbing. Such study could be useful for the designers of tool-machines and for technological engineers when calculating the cutting regime for gears machining.

There are many other works of other authors regarding the dynamics of the hobbing process. The influence of workpiece spindle vibration on hobbing accuracy is studied by Yang Y. (2019).

The torsional elastic angular displacement and its influence on hobbing process is discussed by Hrytsay I. *et al*. (2019).

The errors of the kinematic chain of the gears cutting machines are discussed by Mazuru S. (2019).

The necessity of developing more complex dynamic models for studying the work of mechanisms and machines are shown by Merticaru V., Merticaru E. and Merticaru V.jr. (Merticaru and Merticaru, 2004; Merticaru and Merticaru, 2020; Merticaru, 1991; Merticaru *et al.*, 2009; Merticaru *et al.*, 2014).

2. Kinematic Aspects

The complex gear transmission which is subjected to study is bellow presented in Fig 1.

Such a gear transmission could be used, for example, for manufacturing cylindrical gears by hobbing (*** *Bazele generării suprafețelor*; Iliescu, 2013; Mazuru, 2019; Tabacaru).

The complex gear presented in Fig. 1 is driven by an electric motor which is acting the wheel z_1. The flow of power is divided through gears z_2 and z_4. Then, the flow of power is divided, once again, through wheels z_3' and z_{10}. The wheel z_9 has its motion as a result of linear combination of angular speeds of wheels z_6' and z_7'. Also, the angular speeds of wheels z_9 and z_{11} are correlated.

In the gear drive from Fig. 1, the shaft of the wheel z_9 is the workpiece spindle. The shaft of the wheel z_{11} is the hob spindle.

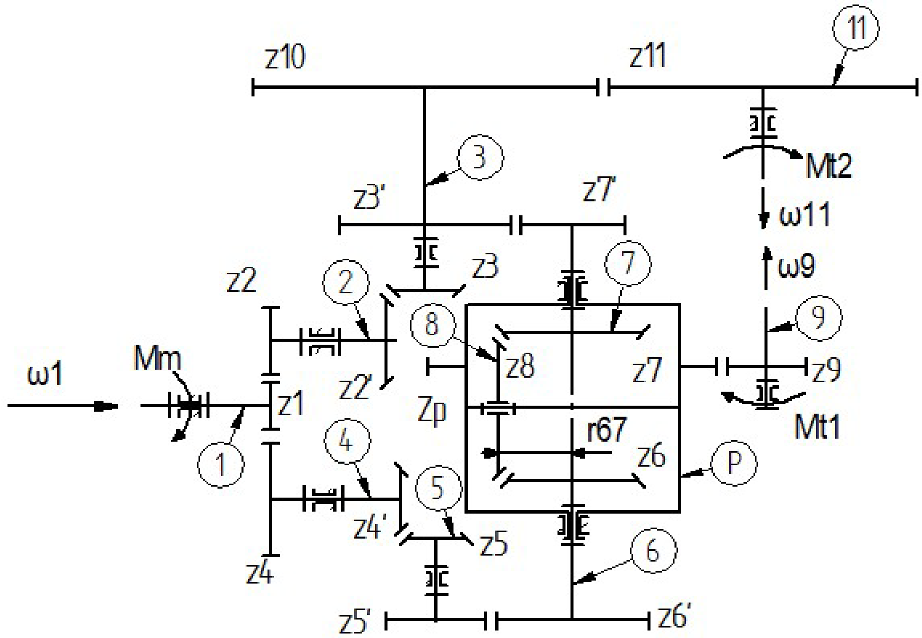

Fig. 1 − The complex gear transmission.

Usually, between the angular speeds ω_9 and ω_{11} there is the following relation (*** *Bazele generării suprafețelor*):

$$\frac{\omega_9}{\omega_{11}} = \frac{k}{z_{ps}} \quad (1)$$

where: k is the number of starts of the hob and z_{ps} is the number of teeth of the workpiece.

The gear in Fig. 1 has 1 degree of mobility. In Fig. 1, M_m is the motor torque. M_{t1} and M_{t2} are technological torques, resulted from the cutting process, acting on wheels z_9 and, respectively z_{11}. Also, z_i represents the number of teeth of wheel "i", ω_i represents the angular speed of wheel "i". Also, we suppose to know the inertia moments J_i of kinematic elements "i", the mass m_8 of satellite wheel z_8, the number n_8 of the satellite wheels z_8, and the radius of disposal r_{67} of the satellite wheels z_8 (shown in Fig. 1).

The transmission error of a gear is defined, as can be seen in Fig. 2 (Raghuraman, 2018; Kučera, 2018; Vispute, 2017; Atanasiu, 2010), by the following Eq. (2):

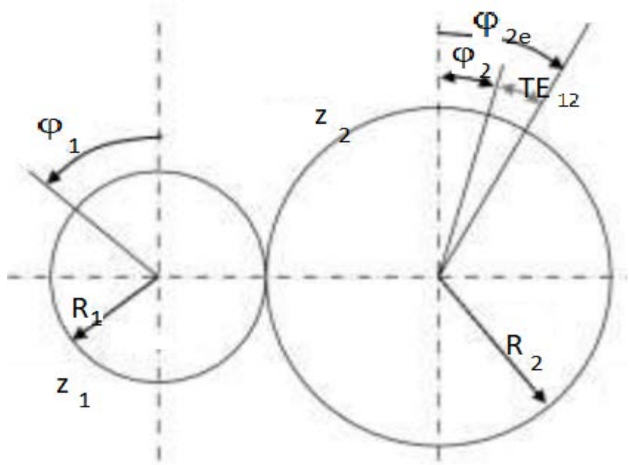

Fig. 2 − The transmission error of a gear.

$$TE_{12} = \varphi_{2e} - \frac{z_1}{z_2} \cdot \varphi_1 \tag{2}$$

where: z_1 is the number of teeth of the pinion, φ_1 is the rotation angle of the pinion, z_2 is the number of teeth of the driven gear, φ_{2e} is the effective rotation angle of the driven gear (affected by the transmission error TE_{12}) and φ_2 is the rotation angle of the driven gear unaffected by TE_{12}:

$$\varphi_2 = \frac{z_1}{z_2} \cdot \varphi_1 \tag{3}$$

The transmission error of a gear depends on various factors such as: gearcase accuracy, gear profile accuracy, teeth spacing error of the gear, gear helix accuracy, quality of the contact surface finish, thermal distortions, gear motion, gearcase distortions, gear distortions, gear teeth deflections (Atanasiu, 2010; Kučera, 2018; Raghuraman, 2018; Vispute, 2017). That is, the transmission error is a function depending on time and angle of rotation:

$$TE_{12} = TE_{12}\left(t, \varphi_1\right) \tag{4}$$

By deriving in relation to time the relation (2), there can be get:

$$\dot{TE}_{12} = \frac{d\left(TE_{12}\right)}{dt} = \omega_{2e} - \frac{z_1}{z_2} \cdot \omega_1 \tag{5}$$

that is,

$$\frac{\omega_{2e}}{\omega_1} = \frac{T\dot{E}_{12}}{\omega_1} + \frac{z_1}{z_2} \tag{6}$$

where ω_1 is the angular speed of the pinion z_1, ω_{2e} is the angular speed of the gear z_2 affected by the transmission error TE_{12}.

Like relation (6), for the complex gear in Fig. 1, the following relations can be written:

$$\frac{\omega_{3e}}{\omega_1} = \frac{T\dot{E}_{2'3}}{\omega_1} + \frac{z_{2'}}{z_3} \cdot \frac{\omega_{2e}}{\omega_1} \tag{7}$$

$$\frac{\omega_{11e}}{\omega_1} = \frac{T\dot{E}_{10-11}}{\omega_1} + \frac{z_{10}}{z_{11}} \cdot \frac{\omega_{3e}}{\omega_1} \tag{8}$$

$$\frac{\omega_{4e}}{\omega_1} = \frac{T\dot{E}_{14}}{\omega_1} + \frac{z_1}{z_4} \tag{9}$$

$$\frac{\omega_{5e}}{\omega_1} = \frac{T\dot{E}_{4'5}}{\omega_1} + \frac{z_{4'}}{z_5} \cdot \frac{\omega_{4e}}{\omega_1} \tag{10}$$

$$\frac{\omega_{6e}}{\omega_1} = \frac{T\dot{E}_{5'6'}}{\omega_1} + \frac{z_{5'}}{z_{6'}} \cdot \frac{\omega_{5e}}{\omega_1} \tag{11}$$

$$\frac{\omega_{7e}}{\omega_1} = \frac{T\dot{E}_{3'7'}}{\omega_1} + \frac{z_{3'}}{z_{7'}} \cdot \frac{\omega_{3e}}{\omega_1} \tag{12}$$

$$\frac{\omega_{pe}}{\omega_1} = \frac{\omega_{6e} + \omega_{7e} - T\dot{E}_{76}}{2 \cdot \omega_1} \tag{13}$$

or

$$\frac{\omega_{pe}}{\omega_1} = \frac{\omega_{6e} + \omega_{7e} - T\dot{E}_{86} - \frac{z_8}{z_6} \cdot T\dot{E}_{78}}{2 \cdot \omega_1} \tag{14}$$

$$\frac{\omega_{8e}}{\omega_1} = \frac{T\dot{E}_{78}}{\omega_1} + \frac{z_7}{z_8} \cdot \frac{\omega_{7e}}{\omega_1} + \frac{\omega_{pe}}{\omega_1} \cdot \left(1 - \frac{z_7}{z_8}\right) \tag{15}$$

$$\frac{\omega_{9e}}{\omega_1} = \frac{T\dot{E}_{p9}}{\omega_1} + \frac{z_p}{z_9} \cdot \frac{\omega_{pe}}{\omega_1} \tag{16}$$

In Eqs. (7-16), $T\dot{E}_{ij}$ is the derivative of the transmission error of the gear "*ij*" and ω_{ie} is the angular speed of the gear "*i*" affected by the transmission errors.

3. Dynamic Aspects

The cutting force when hobbing a gear is variable and quite complicated to calculate.

Many authors have written papers regarding the way to calculate the cutting force using various methods (Bostan *et al.* 2001; Abood, 2002; Bostan *et al.* 2004a; Bostan *et al.* 2004b; Bouzakis *et al.* 2008; Alazar *et al.* 2011; Bostan *et al.* 2011; Bostan *et al.* 2013; Iliescu, 2013; Rusu, 2015; Sabkhi *et al.* 2015; Hrytsay *et al.*, 2019; Mazuru, 2019; Yang, 2019; Tabacaru).

The cutting force depends on the following factors: rotation speed of the hob, geometry of the hob, the material and the dimensions of the workpiece, the cutting regime of the hobbing process.

The technological torque acting on the workpiece can be approximated as following:

$$M_{t1} = C_9 \cdot F_0 + C_9 \cdot F \cdot \sin\left(k_1 \cdot \omega_{11} \cdot t\right) \tag{17}$$

Similarly, the technological torque acting on the hob can be approximated as following:

$$M_{t2} = C_{11} \cdot F_0 + C_{11} \cdot F \cdot \sin\left(k_1 \cdot \omega_{11} \cdot t\right) \tag{18}$$

In Eqs. (17) and (18), C_9, C_{11}, F_0, F and k_1 are quantities that depend on geometry of the hob, the material and the dimensions of the workpiece, the cutting regime of the hobbing process and ω_{11} is the rotation speed of the hob.

If adopting, for the complex gear in Fig. 1, a "reducing element" dynamic model with rotation motion, with the gear z_1 as reducing element, then the differential motion equation for this dynamic model can be written as following:

$$J_{red}\cdot\frac{d\omega_1}{dt}+\frac{\omega_1^2}{2}\cdot\frac{dJ_{red}}{d\varphi_1}=M_{red} \tag{19}$$

In relation (19), J_{red} is the reduced inertia moment of the dynamic model, which can be written as:

$$\begin{aligned}J_{red}&=J_1+J_2\cdot\left(\frac{\omega_{2e}}{\omega_1}\right)^2+J_3\cdot\left(\frac{\omega_{3e}}{\omega_1}\right)^2+J_{11}\cdot\left(\frac{\omega_{11e}}{\omega_1}\right)^2+J_4\cdot\left(\frac{\omega_{4e}}{\omega_1}\right)^2+\\&J_5\cdot\left(\frac{\omega_{5e}}{\omega_1}\right)^2+J_6\cdot\left(\frac{\omega_{6e}}{\omega_1}\right)^2+J_7\cdot\left(\frac{\omega_{7e}}{\omega_1}\right)^2+J_p\cdot\left(\frac{\omega_{pe}}{\omega_1}\right)^2+\\&n_8\cdot J_8\cdot\left(\frac{\omega_{8e}}{\omega_1}\right)^2+n_8\cdot m_8\cdot\left(\frac{\omega_{pe}\cdot r_{67}}{\omega_1}\right)^2+J_9\cdot\left(\frac{\omega_{9e}}{\omega_1}\right)^2\end{aligned} \tag{20}$$

where $\frac{\omega_{ie}}{\omega_1}$ can be calculated with Eqs. (6-16), J_i is the moment of inertia of element "*i*".

The reduced moment M_{red} in Eq. (19) can be written as:

$$M_{red}=M_m-M_{t1}\cdot\frac{\omega_{9e}}{\omega_1}-M_{t2}\cdot\frac{\omega_{11e}}{\omega_1} \tag{21}$$

In Eq. (21), M_m is the motor torque of the motor driving the complex gear. If this motor is an alternative current motor, there can be approximated:

$$M_m=\frac{M_n\cdot(\omega_1-\omega_s)}{\omega_n-\omega_s} \tag{22}$$

where M_n is the nominal torque of the electrical motor, ω_n is the nominal angular speed of the electrical motor, ω_s is the synchronism angular speed of the electrical motor.

Considering the Eqs. (17), (18), (21), (22), there results:

$$M_{red}=\frac{M_n\cdot(\omega_1-\omega_s)}{\omega_n-\omega_s}-\left(C_9\cdot\frac{\omega_{9e}}{\omega_1}+C_{11}\cdot\frac{\omega_{11e}}{\omega_1}\right)\cdot\left[F_0+F\cdot\sin(k_1\cdot\omega_{11}\cdot t)\right] \tag{23}$$

For a specific gear hobbing case, if knowing the transmission errors for all the gears in the mechanism, the driving electric motor and the parameters of the cutting regime of hobbing process, then J_{red} and M_{red} in Eq. (19) can be calculated. Then, the differential motion equation (19), can be numerically integrated, resulting the real angular speed ω_1.

In this way, the real motion of the complex gear depending on the cutting forces and the transmission errors of the gears can be determined.

It can be observed that, due to transmission errors, the reduced moment of inertia J_{red} is not constant,

$$J_{red} = J_{red}\left(\varphi_1\right) \tag{24}$$

Also, the reduced torque M_{red} is a function of time, position and speed:

$$M_{red} = M_{red}\left(t, \varphi_1, \omega_1\right) \tag{25}$$

4. Conclusions

In the paper, a dynamic study, based on researches described in a previous paper of the same authors (Merticaru and Merticaru, 2020), is developed.

The transmission errors of a complex gear and the cutting forces of a gear hobbing process are considered to conceive the dynamic model. The differential equation of motion is written and some discussions are made.

Due to transmission errors, the reduced moment of inertia J_{red} is not constant and also the reduced torque M_{red} is a function of time, position and speed.

The study in the paper might be useful for those who want to design such kind of gear transmissions and to calculate a specific gear hobbing cutting regime, to obtain better quality for machined gears.

Some future research development directions are intended to be followed, such as models validation through computerized modeling and simulation and also through some physical experiments.

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MODELE ALE INFLUENȚELOR ERORILOR DE TRANSMITERE ȘI ALE FORȚELOR DINTR-UN PROCES DE DANTURARE ASUPRA UNEI TRANSMISII COMPLEXE CU ANGRENAJE

(Rezumat)

Lucrarea prezintă un demers teoretic de cercetare care presupune un studiu asupra dinamicii unei transmisii complexe cu angrenaje. Analiza cinematicii ia în considerare erorile de transmisie din angrenajele componente. De asemenea, evaluarea parametrilor dinamici din modelul propus ia în considerare forțele de așchiere dintr-un proces de danturare care poate avea în lanțul cinemtic de acționare o asemenea transmisie complexă. Modelul propus include obținerea ecuației diferențiale a energiei cinetice. Totodată, sunt formulate unele discuții privind mecanismul supus studiului de caz. Practic, sunt propuse ca rezultate livrabile ale studiului, modele ale influențelor erorilor de transmisie din angrenaje și ale forțelor de așchiere la dantuarea prin generare, asupra transmisiei complexe analizate. Sunt formulate, în final, unele concluzii ale studiului. Aspectele analizate și prezentate în lucrare au utilitate pentru proiectarea unor asemenea transmisii complexe și pentru calculul parametrilor unor procese de danturare prin generare.